Coordinate Geometry

Coordinate Geometry, also known as Cartesian Geometry, is a branch of geometry where the position of points on a plane is described using ordered pairs of numbers called coordinates. These coordinates are based on the Cartesian coordinate system, which consists of two perpendicular axes: the horizontal axis (x-axis) and the vertical axis (y-axis). This system is widely used to solve problems involving shapes, distances, midpoints, and areas.

1. Distance Formula

The distance formula is used to determine the straight-line distance between two points (x_1, y_1) and (x_2, y_2) in a Cartesian plane.

Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Explanation:

The formula is derived from the Pythagorean theorem. In the Cartesian plane, the horizontal difference $(x_2 - x_1)$ and the vertical difference $(y_2 - y_1)$ form the two perpendicular sides of a right triangle. The distance between the points is the hypotenuse of this triangle.

Example:

Find the distance between points $(3, 4)$ and $(7, 1)$:

$$d = \sqrt{(7-3)^2 + (1-4)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = 5$$

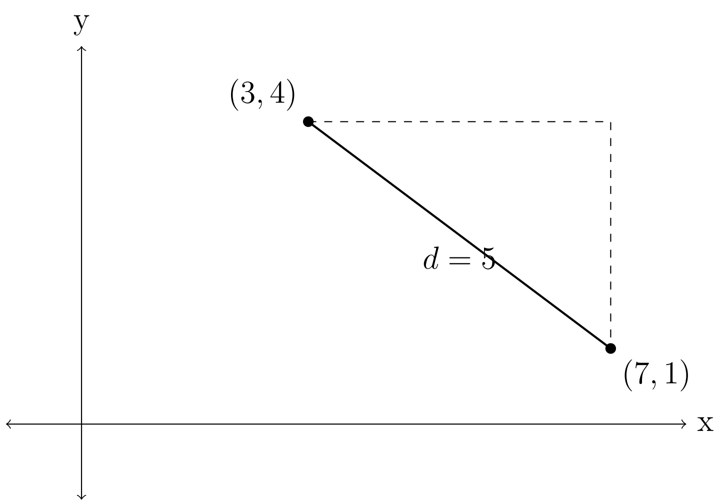

Figure 1: Illustration of the distance between two points

2. Section Formula (Internal Division)

The section formula helps find the coordinates of a point $P(x, y)$ that divides a line segment joining two points (x_1, y_1) and (x_2, y_2) in a given ratio $m : n$ internally.

Formula:

$$x = \frac{mx_2 + nx_1}{m + n}, \quad y = \frac{my_2 + ny_1}{m + n}$$

Explanation:

The coordinates of P are calculated as a weighted average of the coordinates of (x_1, y_1) and (x_2, y_2). The weights are determined by the ratio $m : n$, ensuring that the division point lies within the line segment.

Example:

Find the point dividing the line segment joining $(2, 3)$ and $(6, 7)$ in the ratio $2 : 1$:

$$x = \frac{2(6) + 1(2)}{2 + 1} = \frac{12 + 2}{3} = 4, \quad y = \frac{2(7) + 1(3)}{2 + 1} = \frac{14 + 3}{3} = 5$$

Point P is $(4, 5)$.

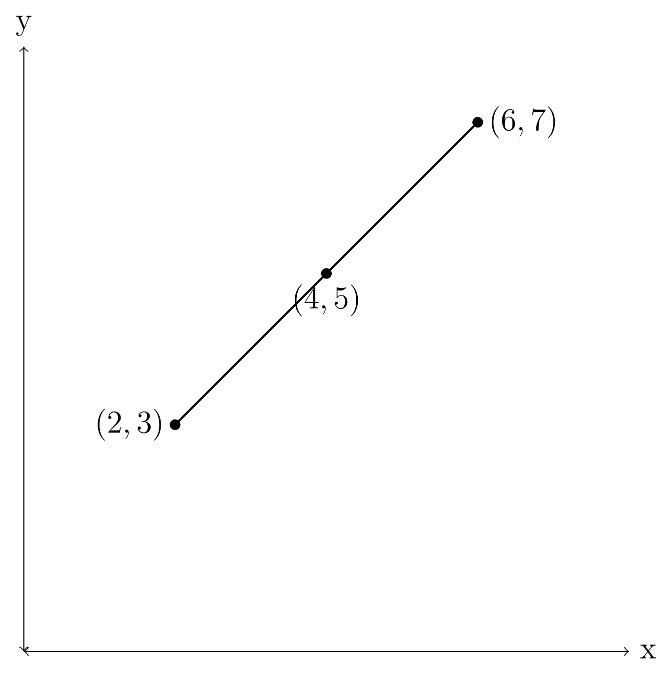

Figure 2: Illustration of section formula with point of division

3. Area of a Triangle

The area of a triangle formed by three points (x_1, y_1), (x_2, y_2), and (x_3, y_3) in a Cartesian plane can be determined using the following formula:

Formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Explanation:

This formula is derived from the determinant method and ensures that the area is always positive, regardless of the order of the points. The absolute value ensures no negative areas.

Example:

Find the area of a triangle with vertices $(1, 2)$, $(4, 6)$, and $(6, 2)$:

$$\text{Area} = \frac{1}{2} |1(6 - 2) + 4(2 - 2) + 6(2 - 6)|$$

$$= \frac{1}{2} |1(4) + 4(0) + 6(-4)| = \frac{1}{2} |4 - 24| = \frac{1}{2} \times 20 = 10$$

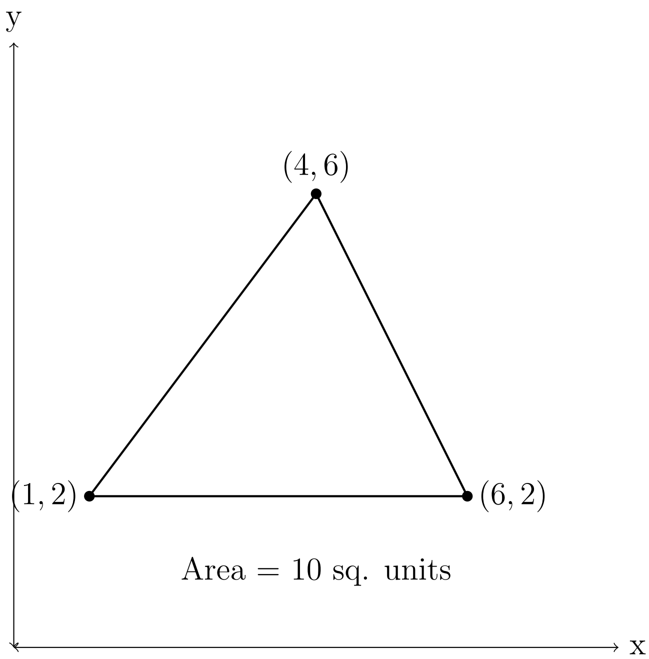

Figure 3: Illustration of triangle area calculation