Comprehensive Algebra Notes for Grade 10

Introduction

This document provides a detailed overview of Algebra topics for Grade 10. Each section includes clear explanations, step-by-step methods, and solved examples to enhance understanding.

1. Polynomials

1.1 Definition of Polynomials

A polynomial is an algebraic expression consisting of variables, coefficients, and non-negative integer exponents.

- General form: $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$
- Examples: $2x^3 - 3x^2 + 5x - 7$, $4y^2 + 6y + 1$
- Non-polynomials: Expressions with negative exponents or variables in the denominator (e.g., x^{-1}, $\frac{1}{x+1}$).

1.2 Types of Polynomials

- **Monomial**: A polynomial with one term (e.g., $3x^2$).
- **Binomial**: A polynomial with two terms (e.g., $x^2 - 4$).
- **Trinomial**: A polynomial with three terms (e.g., $2x^3 - x + 1$).

1.3 Degree of a Polynomial

The degree of a polynomial is the highest power of the variable.

- Example 1: $4x^5 + 3x^3 - 2x + 7$ has a degree of 5.
- Example 2: $3y^4 - 2y^2 + 5$ has a degree of 4.

1.4 Operations on Polynomials

Addition and Subtraction: Combine like terms.

- Example: $(3x^2 + 5x - 2) + (2x^2 - 3x + 7) = 5x^2 + 2x + 5$
- Example: $(4y^3 - y + 6) - (3y^3 + y - 2) = y^3 - 2y + 8$

Multiplication: Use distributive property or expand terms.

- Example: $(x + 2)(x - 3) = x^2 - x - 6$.
- Example: $(2x + 3)(x^2 - x + 1) = 2x^3 + x^2 - x + 3$.

Division: Divide using long division or synthetic division.

- Example: Divide $x^3 + 2x^2 - 3x - 6$ by $x - 1$.

$$\text{Solution: } x^2 + 3x + 1 \text{ with remainder } -5.$$

1.5 Factorization of Polynomials

- **Common Factor Method:**
  - Example: Factorize $3x^2 + 9x$.

$$3x^2 + 9x = 3x(x + 3)$$

- **Grouping Terms:**
  - Example: Factorize $x^3 + 3x^2 + x + 3$.

$$x^3 + 3x^2 + x + 3 = (x^2 + 1)(x + 3)$$

- **Special Formulas:**

$$a^2 - b^2 = (a - b)(a + b)$$
$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

- Example: Factorize $x^2 - 9$.

$$x^2 - 9 = (x - 3)(x + 3)$$

- Example: Factorize $x^3 + 8$.

$$x^3 + 8 = (x + 2)(x^2 - 2x + 4)$$

2. Quadratic Equations

2.1 General Form

A quadratic equation is of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

2.2 Methods of Solving Quadratic Equations

1. **Factorization:** Split the middle term to factorize.

   - Example: Solve $x^2 - 5x + 6 = 0$.

   $$x^2 - 5x + 6 = (x-2)(x-3) = 0$$
   $$\therefore x = 2 \text{ or } x = 3.$$

2. **Completing the Square:** Rewrite the equation as a perfect square trinomial.

   - Example: Solve $x^2 + 6x + 5 = 0$ by completing the square.

   $$x^2 + 6x + 5 = 0$$
   $$(x+3)^2 - 9 + 5 = 0$$
   $$(x+3)^2 = 4$$
   $$\therefore x = -3 \pm 2.$$

3. **Quadratic Formula:** Use $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

   - Example: Solve $2x^2 - 4x - 6 = 0$.

   $$a = 2,\ b = -4,\ c = -6$$
   $$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-6)}}{2(2)}$$
   $$x = \frac{4 \pm \sqrt{16+48}}{4} = \frac{4 \pm \sqrt{64}}{4}$$
   $$x = \frac{4 \pm 8}{4}$$
   $$\therefore x = 3 \text{ or } x = -1.$$

4. **Graphical Method:** To solve a system of linear equations graphically, plot both equations on the same coordinate plane and identify their point of intersection.

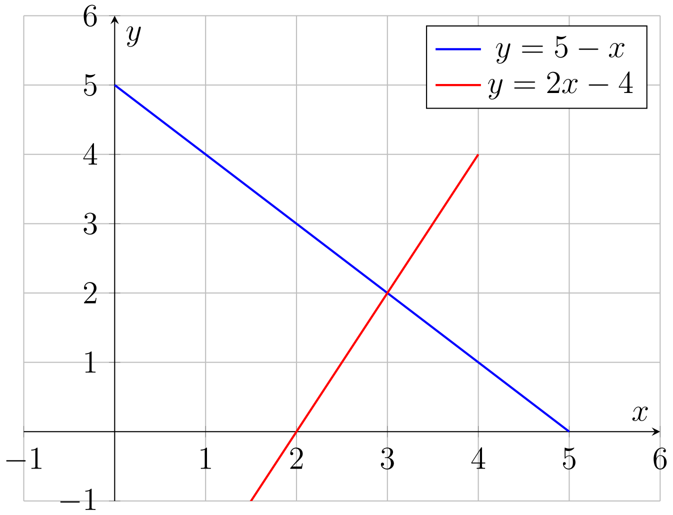
   - Example: Solve $x + y = 5$ and $2x - y = 4$ graphically.

     (a) Rewrite the equations in slope-intercept form:

     $$y = 5 - x \quad \text{(Equation 1)}$$
     $$y = 2x - 4 \quad \text{(Equation 2)}$$

     (b) Plot the equations on the coordinate plane.

     (c) The lines intersect at $(3, 2)$, so the solution is $x = 3, y = 2$.



y
x
y = 5 − x
y = 2x − 4
6
5
4
3
2
1
−1
−1
1
2
3
4
5
6

3. Arithmetic Progressions (AP)

3.1 Definition

An arithmetic progression (AP) is a sequence where each term differs from the previous term by a constant difference d.

- General form: $a, a+d, a+2d, \ldots$.
- Example: $3, 7, 11, 15, \ldots$ with $d = 4$.

3.2 nth Term of an AP

The nth term is given by:

$$a_n = a + (n-1)d$$

- Example: Find the 10th term of the AP $2, 5, 8, \ldots$.

$$a = 2,\ d = 3,\ n = 10$$
$$a_{10} = 2 + (10-1)(3) = 2 + 27 = 29.$$

- Example: Find the 15th term of the AP $7, 13, 19, \ldots$.

$$a = 7,\ d = 6,\ n = 15$$
$$a_{15} = 7 + (15-1)(6) = 7 + 84 = 91.$$

3.3 Sum of First n Terms of an AP

The sum of the first n terms is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

- Example: Find the sum of the first 15 terms of the AP $3, 7, 11, \ldots$.

$$a = 3,\ d = 4,\ n = 15$$
$$S_{15} = \frac{15}{2}[2(3) + (15-1)(4)] = \frac{15}{2}[6 + 56] = \frac{15}{2} \cdot 62 = 465.$$

- Example: Find the sum of the first 10 terms of the AP $5, 10, 15, \ldots$.

$$a = 5,\ d = 5,\ n = 10$$
$$S_{10} = \frac{10}{2}[2(5) + (10-1)(5)] = 5[10 + 45] = 5 \cdot 55 = 275.$$

4. Linear Equations in Two Variables

4.1 General Form

A linear equation in two variables is of the form:

$$ax + by + c = 0$$

where a, b, c are real numbers and $a \neq 0$, $b \neq 0$.

- Example: $3x + 2y - 5 = 0$.
- Example: $2x - y + 4 = 0$.

4.2 Methods of Solving Linear Equations

1. **Substitution Method:** Solve one equation for one variable and substitute into the other.

   - Example: Solve $x + y = 5$ and $2x - y = 4$.

   $$\begin{aligned} &y = 5 - x \\ &2x - (5 - x) = 4 \\ &2x - 5 + x = 4 \\ &3x = 9 \implies x = 3,\ y = 2. \end{aligned}$$

   - Example: Solve $3x + 4y = 10$ and $x - 2y = 1$.

   $$\begin{aligned} &x = 1 + 2y \\ &3(1 + 2y) + 4y = 10 \\ &3 + 6y + 4y = 10 \\ &10y = 7 \implies y = \frac{7}{10},\ x = \frac{17}{10}. \end{aligned}$$

2. **Elimination Method:** Add or subtract equations to eliminate one variable.

   - Example: Solve $2x + 3y = 12$ and $x - y = 3$.

   $$\begin{aligned} &\text{Multiply second equation by 3: } 3x - 3y = 9 \\ &(2x + 3y) + (3x - 3y) = 12 + 9 \\ &5x = 21 \implies x = 4.2,\ y = 2. \end{aligned}$$

   - Example: Solve $x + 2y = 8$ and $3x - 4y = 2$.

   $$\begin{aligned} &\text{Multiply first equation by 2: } 2x + 4y = 16 \\ &\text{Add equations: } (2x + 4y) + (3x - 4y) = 16 + 2 \\ &5x = 18 \implies x = 3.6,\ y = 2.2. \end{aligned}$$